

Update on a short-distance D^0 -meson mixing calculation with $N_f = 2 + 1$ flavors

C.C. Chang^{a,f}, C. Bernard^b, C.M. Bouchard^c, A.X. El-Khadra^{a,f}, E.D. Freeland^d, E. Gámiz^e, A.S. Kronfeld^{f,g}, J. Laiho^h, R.S. Van de Water^f

^aPhysics Department, University of Illinois, Urbana, IL 61801, USA

^bDepartment of Physics, Washington University, St. Louis, MO 63130, USA

^cDepartment of Physics, The Ohio State University, Columbus, OH 43210, USA

^dLiberal Arts Department, The School of the Art Institute of Chicago, Chicago, IL 60603, USA

^eCAFPE and Departamento de Física Teórica y del Cosmos, Universidad de Granada, E-18002 Granada, Spain

^fTheoretical Physics Department, Fermi National Accelerator Laboratory,[†] Batavia, IL 60510, USA

^gInstitute for Advanced Study, Technische Universität München, 85748 Garching, Germany

^hDepartment of Physics, Syracuse University, Syracuse, NY 13244, USA

Fermilab Lattice and MILC Collaborations

E-mail: cchang5@illinois.edu

We present an update on our calculation of the short-distance D^0 -meson mixing hadronic matrix elements. The analysis is performed on the MILC collaboration's $N_f = 2 + 1$ asqtad configurations. We use asqtad light valence quarks and the Sheikoleslami-Wohlert action with the Fermilab interpretation for the valence charm quark. $SU(3)$, partially quenched, rooted, staggered heavy-meson chiral perturbation theory is used to extrapolate to the chiral-continuum limit. Systematic errors arising from the chiral-continuum extrapolation, heavy-quark discretization, and quark-mass uncertainties are folded into the statistical errors from the chiral-continuum fits with methods of Bayesian inference. A preliminary error budget for all five operators is presented.

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1. Introduction

D^0 -meson mixing is currently the least well understood meson mixing process. Experimental efforts underway or planned at LHCb, BES III, and Belle II, should improve our understanding and ignite excitement for the future of charm physics. In the Standard Model (SM), the short-distance contributions to D^0 -meson mixing are GIM suppressed by $m_s^2 - m_d^2$ and Cabbibo suppressed by $|V_{ub}V_{cb}^*|^2$; therefore D^0 -meson mixing is expected to receive significant contributions from the long-distance processes in the Standard Model. However, it is also possible for D^0 -meson mixing to receive enhancements from short-distance new physics (NP) contributions. Therefore, in conjunction with next generation flavor factories, knowledge of the five short-distance hadronic matrix elements will allow for model-discrimination between NP theories [1]. The short-distance matrix elements are described by a basis of five 4-quark operators that are invariant under Lorentz, Fierz, charge conjugation, parity inversion, and time reversal transformations and may be expressed as the following:

$$\mathcal{O}_1 = \bar{c}^\alpha \gamma^\mu L u^\alpha \bar{c}^\beta \gamma^\mu L u^\beta, \quad \mathcal{O}_2 = \bar{c}^\alpha L u^\alpha \bar{c}^\beta L u^\beta, \quad \mathcal{O}_3 = \bar{c}^\alpha L u^\beta \bar{c}^\beta L u^\alpha, \quad (1.1)$$

$$\mathcal{O}_4 = \bar{c}^\alpha L u^\alpha \bar{c}^\beta R u^\beta, \quad \mathcal{O}_5 = \bar{c}^\alpha L u^\beta \bar{c}^\beta R u^\alpha, \quad (1.2)$$

where L and R are the left- and right-handed projection operators, while c and u denote the charm and up quarks respectively. The operators in Eq. (1.2) couple to right-handed quarks and, therefore, appear only in NP scenarios.

2. Lattice setup and correlator analysis

The correlators pertinent to this project are constructed on a large subset of the MILC gauge configurations [2] with 2+1 asqtad staggered sea quarks. A complete list of ensembles used for this project is given in Ref. [3]. The light valence quarks are also generated with the asqtad action, with masses ranging from m_s to $m_s/20$. We have a large range of valence masses, hence we use partially quenched chiral perturbation theory to extrapolate to physical up- and down-quark masses. For the heavy charm quark, we use the Sheikoleslami-Wohlert action with the Fermilab interpretation, which ensures that the couplings in the theory are smoothly bounded for $am_q \ll 1$, as well as in the limit $am_q \rightarrow 0$, resulting in well controlled errors. The heavy-quark Lagrangian is tree-level improved and the lattice operators corresponding to the \mathcal{O}_i use rotated heavy-quark fields, resulting in errors starting at $\mathcal{O}(\alpha_s a, a^2)$.

Results of the correlator analysis have been presented in Ref. [3] and are complete for all five 4-quark operators. Under renormalization, the sets of operators given in Eq. (1.1) and (1.2) mix among each other. Thus,

$$\langle D^0 | \mathcal{O}_i | \bar{D}^0 \rangle^{\overline{\text{MS}}-\text{NDR}}(m_c) = \sum_{j=1}^5 [\delta_{ij} + \alpha_s(q^*) \zeta_{ij}^{\overline{\text{MS}}-\text{NDR}}(am_c) + \mathcal{O}(\alpha_s^2(q^*))] \langle D^0 | \mathcal{O}_j | \bar{D}^0 \rangle^{\text{lat}}. \quad (2.1)$$

The $\zeta_{ij}^{\overline{\text{MS}}-\text{NDR}}$ s are matching coefficients relating one-loop lattice and continuum renormalizations

evaluated at the charm quark mass m_c , and $\alpha_s(q^*)$ is the strong coupling discussed in Ref. [5]. The one-loop continuum calculations require choosing an additional set of evanescent operators during intermediate steps of dimensional regularization. We report results using the BBGLN [6] scheme. Once the analysis has been finalized, results in the BJU [7] scheme will also be reported. The errors from renormalization and matching start at $\mathcal{O}(\alpha_s^2)$, as suggested by Eq. (2.1). For brevity, we will use the short-hand $\langle \mathcal{O}_i \rangle \equiv \langle D^0 | \mathcal{O}_i | \bar{D}^0 \rangle^{\overline{\text{MS}}-\text{NDR}}(m_c)$ when referring the renormalized matrix element below.

The charm quark mass is set by tuning the D_s -meson mass to its physical value [8, 9]. Corrections to the slight mistunings are implemented by linearly extrapolating the matrix element to the correct (tuned) charm mass m_c ,

$$\langle \mathcal{O}_i \rangle_{\text{tune}} = \langle \mathcal{O}_i \rangle + \sigma_i \Delta(1/M_2), \quad (2.2)$$

where σ_i is the slope of $\langle \mathcal{O}_i \rangle$ with respect to the inverse kinetic mass $1/M_2$ and is obtained by performing a correlated unconstrained fit on the $a \approx 0.12$ fm, $m_l/m_s = 0.2$ ensemble at two valence mass points and two values of m_c . The errors of the corrections to the heavy-quark mistuning are determined by the precision of the linear fits performed to extract σ_i as well as the determination of the tuned m_c , outlined in Refs. [8, 9].

3. Chiral-continuum extrapolation

To extrapolate to the chiral-continuum limit, we use SU(3), partially quenched, rooted, staggered, heavy-meson chiral perturbation theory [12]. For reviews of heavy meson and staggered chiral perturbation theory see: [11, 10]. The expression has the schematic form,

$$\langle \mathcal{O}_i \rangle = \beta_i \left(1 + \frac{\mathcal{W}_{u\bar{c}} + \mathcal{W}_{c\bar{u}}}{2} + \mathcal{T}_u^{(i)} + \frac{C(\beta_{j \neq i})}{\beta_i} \tilde{\mathcal{T}}_u^{(i)} + \text{analytic terms} \right) + \beta'_i \left(\mathcal{Q}_u^{(i)} + \frac{D(\beta'_{j \neq i})}{\beta'_i} \tilde{\mathcal{Q}}_u^{(i)} \right). \quad (3.1)$$

The β s and β' s along with the coefficients in the analytic terms are the low energy constants (LECs) of the theory and are determined from fits to the matrix element data. The functions C and D are linear in $\beta_{j \neq i}$ for each $\langle \mathcal{O}_i \rangle$, introducing mixing between the leading-order LECs within the sets $\{\langle \mathcal{O}_1 \rangle, \langle \mathcal{O}_2 \rangle, \langle \mathcal{O}_3 \rangle\}$ and $\{\langle \mathcal{O}_4 \rangle, \langle \mathcal{O}_5 \rangle\}$. The terms \mathcal{W} , \mathcal{T} and \mathcal{Q} denote the chiral logarithms arising from the wavefunction renormalization, tadpole, and sunset one-loop Feynman diagrams. Using staggered light quarks and local (not point-split) operators introduces wrong-spin taste-mixing chiral logarithms $\tilde{\mathcal{T}}$ and $\tilde{\mathcal{Q}}$, however these contributions do not introduce new LECs, as indicated by the C and D functions in Eq. (3.1). From examining the correlator fits, we observe the β s to be of order 1. The β s are introduced into the fit via priors and are loosely constrained with a prior width of 10, such that the data determine their values.

Analytic terms in the chiral fit capture the effects of explicit NLO SU(3) flavor symmetry breaking and SU(4) taste breaking as well as NNLO contributions. The dependence of the valence mass, sea mass and taste breaking effects are parameterized by dimensionless “natural χ PT”

parameters, which yield coefficients that are naturally of $O(1)$ [13],

$$x_{u,l,s} \equiv \frac{(r_1 B_0)(r_1/a)(2am_{u,l,s})}{8\pi^2 f_\pi^2 r_1^2} \quad x_{\bar{\Delta}} \equiv \frac{r_1^2 a^2 \bar{\Delta}}{8\pi^2 f_\pi^2 r_1^2} \quad (3.2)$$

where $m_{u,l,s}$ corresponds to the valence up, sea up/down and sea strange masses and $\bar{\Delta}$ is the average taste splitting. The NLO and NNLO analytic terms are,

$$\text{NLO analytic} = c_0 x_u + c_1 (2x_l + x_s) + c_2 x_{\bar{\Delta}} \quad (3.3)$$

$$\text{NNLO analytic} = \sum_j d_j F_j(x_n x_m) \quad (3.4)$$

where the NLO coefficients c_i are loosely constrained while the NNLO coefficients d_j are constrained to be $O(1)$. The functions $F_j(x_n x_m)$ represents the set of quadratic functions in $x_{u,l,s,\bar{\Delta}}$. Our fits results are insensitive to the addition of terms beyond NNLO.

The largest $1/M_D$ corrections from heavy-meson χ PT arise from the spin splittings (e.g., $M_{D^*} - M_D$) and flavor splittings (e.g., $M_{D_s} - M_D$) and are accounted for in the chiral logarithms presented in Eq. (3.1).

For LECs that cannot be determined by the data, priors are used to constrain the parameters. The largest parametric uncertainty that enters the chiral-continuum extrapolation is the heavy-meson coupling and is accounted for through the prior $g_{D^* D \pi} = 0.53 \pm 0.08$ [14]. Other parameters such as the hyperfine splitting Δ^* is determined by experimental results [15] and are introduced as priors to incorporate experimental uncertainties. Systematic errors arising from the free parameters of the effective theory are all accounted for in the chiral fits.

Along with the chiral-continuum extrapolation, we fold in the $O(\alpha_s a, a^2)$ heavy-quark discretization errors arising from the Lagrangian and operator. We estimate the contributions arising from perturbative matching between continuum QCD and lattice QCD through HQET [4, 16],

$$\mathcal{L}_{\text{QCD}} \doteq \mathcal{L}_{\text{HQET}} = \sum_k C_k^{\text{cont}}(m_c) \mathcal{O}_k, \quad (3.5)$$

$$\mathcal{L}_{\text{lat}} \doteq \mathcal{L}_{\text{HQET}(m_0 a)} = \sum_k C_k^{\text{lat}}(m_c, m_0 a) \mathcal{O}_k, \quad (3.6)$$

from which it follows that the error from each term is

$$\text{error}_k = \left| [C_k^{\text{lat}}(m_c, m_0 a) - C_k^{\text{cont}}(m_c)] \langle \mathcal{O}_k \rangle \right|. \quad (3.7)$$

The $O(a\alpha_s, a^2)$ discretization effects are included as part of the chiral-continuum fit. The mismatch functions are discussed in detail in Ref. [13]. Their coefficients introduced as priors with central values of 0, and widths determined by power counting and are $O(1)$.

4. Error budget

The stability of the chiral-continuum extrapolation is demonstrated in Fig. 1. The preferred fit (blue boxes) is a simultaneous fit over the operators that mix in the χ PT over four lattice spacings, including $O(a\alpha_s, a^2)$ heavy-quark discretization corrections and NNLO analytic terms. The first

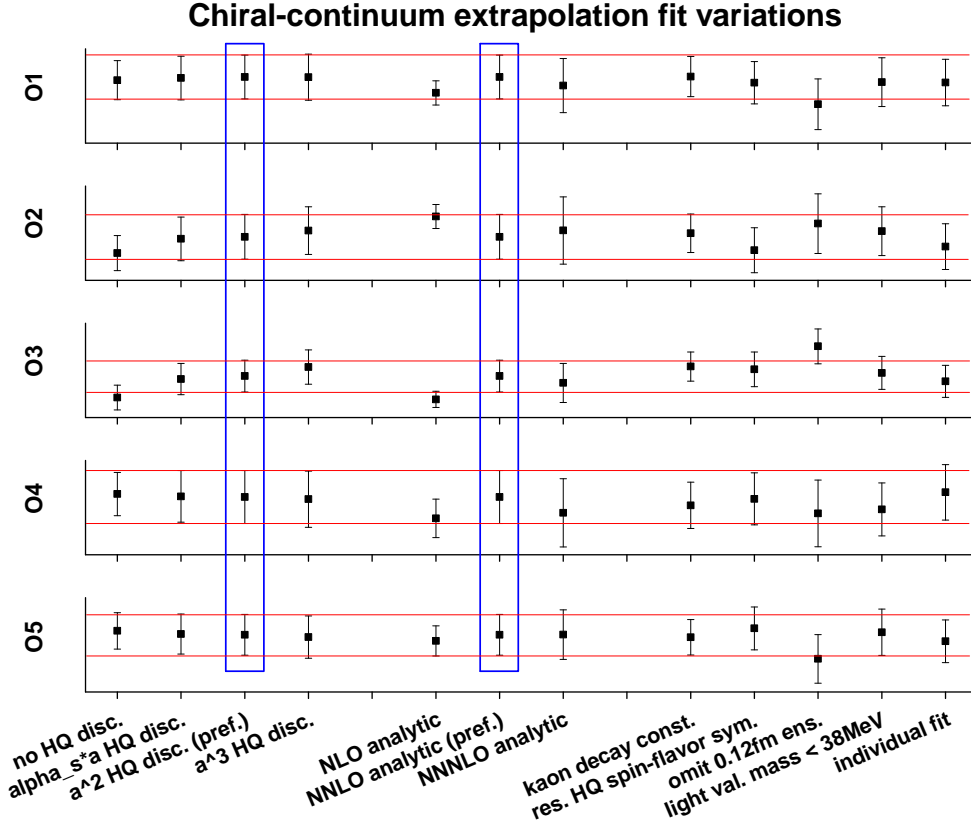


Figure 1: Fit variations for the chiral-continuum extrapolation. The preferred fit is indicated by the blue boxes, with 1σ error bands from that fit indicated by the horizontal red lines. The first group of fits explores options for including heavy-quark discretization errors. The second group explores adding chiral analytic terms. The last group explores parameter and data set changes.

group of four fits in Fig. 1 progressively introduces heavy-quark discretization terms to the fit. The second group of three fits progressively includes more chiral analytic terms. In both cases, the preferred fit lies in the region of stability. The largest changes in the central value occur when introducing the $O(\alpha_s a)$ heavy quark discretization errors, and NNLO analytic terms, showing that truncation errors of the respective expansions are included as part of the preferred fit.

The third set of fits differ from the preferred fits as follows: changing f_π to f_K , restoring heavy quark flavor-spin symmetry, omitting the 0.12 fm ensembles, omitting larger ($> 38\text{MeV}$) light valence quark masses, and performing fits individually for each operator. The fits are stable within one standard deviation.

By incorporating the heavy-quark discretization errors into the chiral-continuum fit, the associated relative error ranges from 2.3–2.9%. We estimate the renormalization error by setting the α_s^2 two-loop coefficients to 1 and using an average value of α_s^2 across all four lattice spacing. This suggests a systematic error of 6.5%. Based on previous D -meson decay constant analysis [13], we expect the finite volume effects to be $< 1\%$.

	$\langle \mathcal{O}_1 \rangle$	$\langle \mathcal{O}_2 \rangle$	$\langle \mathcal{O}_3 \rangle$	$\langle \mathcal{O}_4 \rangle$	$\langle \mathcal{O}_5 \rangle$
Statistical	3.2%	2.1%	3.3%	2.2%	3.9%
Chiral extrapolation	2.2%	2.1%	2.4%	2.1%	3.0%
$(\alpha_s a, a^2)$ HQ error	2.4%	2.4%	2.9%	2.3%	2.7%
HQ mass tuning	0.8%	1.1%	1.0%	1.2%	1.3%
Renormalization	6.5%				
Finite volume	< 1%				
Total error	8.0%	7.7%	8.3%	7.7%	8.7%

Table 1: Preliminary error budget for D^0 -meson mixing hadronic matrix elements in the continuum and for physical quark masses. Values are percent relative errors.

5. Conclusions and outlook

Our chiral-continuum analysis of the D^0 -meson hadronic matrix elements, including a complete error budget, is near completion. Due to the large contribution of the renormalization error, a partially nonperturbative approach to determining the renormalization coefficients is currently being investigated. The results of the matrix elements will also be combined with our decay constants calculated separately [17, 18] and bag parameters will be reported.

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